Swami Ramanand Teerth Marathwada University nanded
Maharashtra, India.

Study of various methods and integral transforms used to solve fractional differential equations

Research proposal

In the subject

Mathematics

Under the faculty of

Science and technology

Submitted by

Miss. Jadhav Shital Babanrao

N.E.S. Science college ,sneh nagar,

Nanded Dist. Nanded,india

shitaljadhav855@gmail.com

Under the supervision of

Dr. R. N. Ingle

Department of mathematics

Bahirji Smark Mahavidyalay, Basmatnagar

431513,(MH), India.

Introduction:

The fractional calculus is a name for the theory of integrals and derivatives of arbitrary order, which unify and generalize the notion of integer-order differentiation and n-fold integration. Fractional calculus was a bright idea of Gottfried Leibniz originating in the end of the sevententeen century. These are referred to as fractional integrals and derivatives, which can be of real or complex order, As a result, integer orders are also included. The applications of fractional calculus are very wide nowadays. The subject of fractional calculus has applications in diverse and widespread fields of engineering and science such as for electromagnetics, viscoelasticity, fluid mechanic electrochemistry, biological population models etc. The fractional derivative models are used for accurate modeling of those systems that require accurate modeling of damping.

Indeed, it is generally understood that the introduction of fractional derivatives provides the system with a critical memory mechanism. It is therefore important to understand the advantages of using fractional derivatives in classical equations. Currently, several different methods and integral transforms are used to solve fractional differential equations, such as classical Laplace and Fourier transform for linear fractional equations[1] , Mellin transform[1], Kamal transform[6], Adomian decomposition method [3], recently the Homotopy Perturbation Method [7], Elzaki transform[16] ,conformal fractional derivative[4], Variation iteration method [10] , New iteration method [2] ,New integral transform [12], Sawi decomposition method[19] which gives approximate and exact solutions for linear and nonlinear fractional differential equation

Definition 1

The Sawi Transform (ST) is obtained over the set of functions as [19,20]:

$$A = \{(t) | \exists M , k_1 > 0, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}} , , if\ t \in (-1)^j \times [0, \infty)\}$$

The ST of f(t) is defined as,

F(s) = S{f(t)}=$\frac{1}{s^2}\int_0^\infty e^{\frac{-t}{s}} f(t)\,dt$, t>0 and s>0

Definition 2

The Kamal transform (KT) is obtained over the set of functions as [11]:

$A = \{(t) | \exists M, k_1 > 0, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0, \infty)\}$

The KT of is defined as,

F(s)= K{f(t)} =$\int_0^\infty e^{\frac{-t}{s}} f(t)\,dt$, t>0, s>0

Review of literature:

Adomian : The author introduced decomposition method called Adomian decomposition method ,which is an effective procedure for solution of nonlinear and/or stochastic continuous-time dynamical systems without usual restrictive assumptions. This method is intended as a convenient tutorial review of the method. [3]

Elzaki: Tarig Elzaki introduced new integral transform called “Elzaki Transform” to facilitate the process of solving ordinary and partial differential equations in time domain [16].

Sharma And Khapre: In this literature, we observe that there are numerous of integral transforms widely used in physics astronomy as well as in engineering. In order to solve differential equation, the integral transform were extensively used. The authors contributes there works on the theory and application of the integral transform such as Laplace, Fourier-Mellin & Hankel, Cosine and Sine transform.[17,18]

Ji-Huan He : The author gives contribution to solve mechanics problems by using different methods. He solved many mechanical problems using variational iteration method, it includes fluid mechanics problems, delay differential equation. Also author gives variational iterature approach to

2-spring system and a new approach to non-linear partial differential equation.[7,10]

Daftardar-Gejji: The author main focus is on fractional order dynamical systems and developing numerical methods to solve fractional differential equations. The author and Jafari introduced new method to solve fractional differential equations called new iterative method. [2,5,8,9]

Fractional Differential transform method(FDTM): FDTM is used to solve frational differential equations. Studies were conducted about solutions of systems of fractional differential equations, system of fractional partial differential equations, fractional order integro-differential equations using fractional differential transform method.[13,14,15]

Conformable Fractional Differential Transform Method (CFDTM): CFDTM is used to solve conformable fractional derivative. CFDTM formulizes conformable fractional power series in a similar manner that FDTM formulizes fractional power series.[21,22]

Objectives:

1) To study basic concepts of fractional calculus.

2) To study various methods used to solve fractional differential equations.

3) To study the different integral transforms used to solve fractional differential equations.

4) To analyze the development in results for fractional differential equation.

5) To develop new results or techniques and integral transforms to solve fractional differential equations.

Research methodology:

The following steps will be including in the methodology:

1) To refer the relevant reference books on fractional calculus.

2) To study different mathematical software such as Mathematica, MATLAB, Latex.

3) To study different methods and integral transform of fractional differential equations.

4) To analyze some papers related to topic.

5) To collect different results and papers from other universities and colleges.

6) To represent papers in conferences and seminars.

7) To represent Numerical solutions of fractional differential equations graphically by Mathematica, MATLAB and observe their nature.

National and international status:

Fractional calculus, which involves the integration and differentiation of an arbitrary or fractional order. Calculus is applied in many areas of life. In physics, engineering, biology, and economics, fractional calculus is becoming a valuable and important tool for analysing scientific problems.

a) International Status: Many scientists and researchers contributed in the development of fractional differential equations and Fractional calculus like, Leibniz, Euler, Lagrange, Laplace, Fourier, Abel, Liouville, Grunwald, Riemann, Riesz, Miller, Ross, Igor Prodlubny, Richtmeyer, Morton, Smith, Anderson, Roache,Lynn R. Willams (1979) Nguyen HuuDenn (1993).

b) National Status: Many Indian scientists and researchers are also contributed remarkable results in the development of fractional calculus

like R.K. Sexana, V.Srivastava, K.N. Rai, S. Das, K. Gupta, S. Saha Roy, V. Daftardar-Gejji, S. Bhalekar, Hossein Jafri, D. Bahuguna, Aalam, , Sunny Chauhan and Pant.

Year wise plan of work:

Sr. no.	**period**	**Plan of work**	**Targets to be achieved**
1	First year	1) study the basic literature regarding research topic. 2) solve some topic related research papers.	Understanding of the subject and solve some research papers
2	Second year	1) Collecting and analyzing research related topics. 2) Discussion of new ideas related to research topic with other researchers and attendance of workshops, conferences.	Attempt to obtain fresh results based on existing work and analyze it.
3	Third year	Publishing some papers and submitting the thesis	Publishing papers and submitting thesis.

References:

[1] Podubny , I. (1999) Fractional Differential Equations. Academics press, New York.

[2] S. Bhalekar, V Daftardar–Gejji Solving evolution equations using a new iterative method. Numer. Methods, partial differential equations. 26(4)(2010)906 − 916.

[3] G. Adomain,(1990) A review of the decomposition method and some recent results for nonlinear equations.

[4] Khalil, R., et al. ”A new definition of fractional derivative.” Journal of Computational and Applied Mathematics 264(2014) : 65 – 70.

[5] S. Bhalekar, V Daftardar–Gejji Solving evolution equations using a new iterative method. Numer. Methods, partial differential equations. 26(4)(2010)906 − 916.

[6] Abdelilah Kamal. H. Sedeeg, The New integral Transform Kamal transform ,Adv. Theor. Appl. Math. 11(4) (2016). 451-458.

[7] J.-H. He, “Homotopy perturbation technique,” Computer Methods in Applied Mechanics and Engineering, vol. 178, no. 3–4, pp. 257-262, (1999).

[8] Daftardar–Gejji v.,Jafari H. An iterative method for solving non linear functional equations. J. Math.Anal. 316(2006) 753-763.

[9] S. Bhalekar, V Daftardar–Gejji proving Convergence of the New Iterative Method,(submitted to Comput. Math. Appl.)

[10] J.H. He, Variational iteration method for delay differential equations, Commun. Non-linear Sci. Numer. Simulation 2 (4) (1997) 235-236.

[11] H. Kamal, A. Sedeeg, 2016. The New İntegral Transform Kamal Transform,Adv Theoret Appl Mathe;11(4):451-8.

[12] H. Jafari, A new general integral transform for solving integral equations. Journal of advanced Research, 2020.

[13] Arikoglu, Aytac, and Ibrahim Ozkol. "Solution of fractional differential equations by using differential transform method." Chaos, Solitons & Fractals 34.5 (2007): 1473-1481.

[14] Ertürk, Vedat Suat, and Shaher Momani. "Solving systems of fractional differential equations using differential transform method." Journal of Computational and Applied Mathematics 215.1 (2008): 142-151. 14

[15] Secer, Aydin, Mehmet Ali Akinlar, and Adem Cevikel. "Efficient solutions of systems of fractional PDEs by the differential transform method." Advances in Difference Equations 2012.1 (2012): 1-7.

[16] T. M. Elzaki, The New İntegral Transform Elzaki Transform Global Journal of Pure and Applied Mathematics ISSN 0973-1768 Volume 7, Number 1 (2011), pp. 57–64.

[17] Sharma, V.D. and Khapre S.A., “Applications on Generalized Two-Dimensional Fractional Cosine Transform,” Int. Journal of Engineering and Innovative Technology (IJEIT), Vol.3, Issue 4, pp. 139-143, October 2013.

[18] Sharma, V.D. and Khapre, S.A., “Some examples on generalized two-dimensional fractional cosine transform in the range $-\infty$ to $+\infty$,” International Journal of science and research, Vol. 3, Issue 8, pp. 1199-1202, August 2014.

[19]M. Higazy, S. Aggarwal, T. A. Nofal, Sawi decomposition method for volterra integral equation with application. Hindawi Journal of Mathematics Volume 2020, Article ID 6687134, 13 pages.

[20] M. M. Abdelrahim Mahgoub, The new integral transform sawi transform.Adv Theoretical Applied Mathematics 2019;14(1):81-7.

[21] Emrah Unal, Ahmet Gokdogan, Solution of Conformable Fractional Ordinary Differential Equations Via Differential Transform Method, Optik, (2017), 264-273.

[22] Eslami M., Taleghani S. A., Differential transform method for conformable fractional partial differential equations, Iranian Journal of Numerical Analysis and Optimization, Vol. 9, No. 2 (2019), 17-29.

Miss. Jadhav Shital Babanrao — Research Scholar

Dr.Rajkumar N. Ingle — Research Guide